Robustness of Morse decompositions for multivalued dynamical systems

Rubén Caballero∗, Alexandre N. Carvalho†, Pedro Marín-Rubio‡ and José Valero §

One of the main goals of the theory of dynamical systems is to characterize the structure of global attractors. It is possible to find sundry literature about this problem; however, it has been recently when papers have been published in which new results and properties of multivalued dynamical systems are presented.

In this sense, the theory of Morse decomposition plays an important role. In fact, the existence of a Lyapunov function, the property of being a dynamically gradient semiflow and the existence of a Morse decomposition are shown to be equivalent in [1].

In this case, we consider a complete metric space $(X, d)$, a family $R \subset C(R_+; X)$, $P(X)$ (the class of nonempty subsets of $X$) and the multivalued map $G : R_+ \times X \to P(X)$; $G(t, u_0) = \{ u(t) : u(\cdot) \in R, u(0) = u_0 \}$.

Moreover, given four properties of the set $R$ and assuming also certain conditions, a robustness result with respect to approximations is obtained; i.e., a dynamically gradient multivalued semiflow is stable under perturbations.

Subsequently, we apply this approximation result to the following family of Chafee-Infante equations:

\[
\begin{align*}
\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} &= f_\varepsilon(u), \quad t > 0, \quad x \in (0, 1), \\
u(t, 0) &= 0, \quad u(t, 1) = 0, \\
u(0, x) &= u_0(x),
\end{align*}
\]

where $\varepsilon > 0$ is a small parameter and $f_\varepsilon$ satisfies:

1. $f_\varepsilon \in C(\mathbb{R})$ and is non-decreasing;
2. $f_\varepsilon(0) = 0$;
3. $f'_\varepsilon(0) > 0$ exists, is finite and $f'_\varepsilon(0) \to +\infty$, as $\varepsilon \to 0^+$;
4. $f_\varepsilon$ is concave if $u > 0$ and convex if $u < 0$;
5. $-1 < f_\varepsilon(s) < 1$, for all $s$, and 
   \[|f_\varepsilon(s) - H_0(s)| < \varepsilon, \text{ if } |s| > \varepsilon,\]
   where
   \[
   H_0(u) = \begin{cases} 
   -1, & \text{if } u < 0, \\
   [-1, 1], & \text{if } u = 0, \\
   1, & \text{if } u > 0,
   \end{cases}
   \]
   is the Heaviside function.

References


∗Departamento de Estadística, Matemáticas e Informática, Universidad Miguel Hernández de Elche, 03202-Elche, Alicante (SPAIN). Email: ruben.caballero@umh.es
†Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Campus de São Carlos, Caixa Postal 668, 13560-970 São Carlos, SP (BRAZIL). Email: andcarva@icmc.usp.br
‡Dpto. Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla, Apdo. de Correos 1160, 41080-Sevilla (SPAIN). Email: pmr@us.es
§Centro de Investigación Operativa, Universidad Miguel Hernández de Elche, 03202-Elche, Alicante (SPAIN). Email: jvalero@umh.es